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Résumé. Cet article présente un modèle de programmation stochastique pour le placement stratégique d'offres dans un marché déréglementé d'électricité. Le modèle est développé pour un producteur d'hydroélectricité "preneur de prix" qui doit gérer une série de centrales sur une même rivière. Le but est de maximiser le profit du producteur en tenant compte de plusieurs contraintes physiques, opérationnelles et financières. Des résultats numériques basés sur des données historiques montrent la supériorité de ce modèle sur un autre modèle qui n'intègre pas de stratégies d'offre.

Mots clés. Hydroélectricité, marché d'énergie, preneur de prix, stratégies d'offre, prix de marché, programmation stochastique.

Abstract. This paper presents a stochastic programming model for strategic bidding in a deregulated energy market. The model is developed for a hydroelectricity price-taker producer managing a series of power plants along a river. The goal is to maximize the profit of the hydroelectricity producer under various physical, operational and financial constraints. Numerical results based on historical data show the superiority of this model over another model that does not integrate the bidding strategy.

Keywords. Hydroelectricity, energy market, price-taker, bidding strategy, market clearing price, stochastic programming.

1 Introduction

Recently, many countries have deregulated their electricity market. As a consequence, new challenges now appear for hydroelectricity producers who need to develop bidding strategies for such markets (i.e., the price and the quantity of electricity to be offered over different periods of the planning horizon).

The hydroelectricity production problem under study involves both hydrological and energy market issues. Hydrological issues relate to the management of the water at hydroelectric facilities. Energy market issues are concerned with bidding strategies, based on the market's price forecasting and price setting mechanisms. The original contribution of this article is thus the integration of bids into a production planning model for a complete hydrological system with a series of facilities along a river. Clearly, the bidding process introduces an additional level of complexity into the problem. But, a clever bidding strategy can allow electricity to be sold when the prices are relatively high, thus increasing the profit margin of the producer.

1.1 Hydrological model

In the application under study, four important sites are found along a river. The first site, P_0, is a dam that retains water from a big reservoir just behind it. The three other sites, P_1, P_2 and P_3, each have an hydroelectric power plant, a dam and a reservoir. Each power plant produces electricity using many turbines that are run independently. The sites are connected in sequence, namely (P_0, P_1, P_2, P_3). Hence, the water released at the upstream dam of site P_i goes into the reservoir of the downstream site P_{i+1}. This simple organization is illustrated in Figure 1. In this figure, a triangle stands for a reservoir with a dam and a little square represents a power plant.

The time taken by the water flow to go from one site to the next is referred to as the "river routing effect". This complex phenomenon can be approximated through river routing coefficients. The latter corresponds to the fraction of water released upstream that arrives at the downstream site every hour after the release. Natural inflows must also be taken into account, like those coming from snow melting, rain, runoff water and natural river flow (for the head reservoir). These inflows are stochastic, but are handled here in a deterministic way, by using mean hourly values provided by a natural inflow forecast model.

To avoid structural damage, the water level at each dam must also lie between a minimum and a maximum value, which are the same all year round. Each dam possesses a mechanism to spill a large quantity of water, if necessary. The spill flow is controlled quite precisely by adjusting gate openings. However, spilling should be avoided as much as possible, given that no electricity is produced in this case.

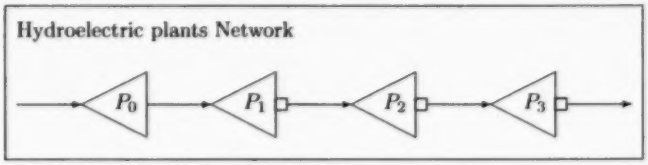

Figure 1: Hydroelectric Network

The amount of electricity produced by each turbine (unit) is a function of the water head, unit flow and unit type. These curves are determined empirically and have, in general, a polynomial form. When a unit is started, it suffers some wear because of the huge water pressure applied to it. So, there is a cost associated with starting a unit, and this cost is also taken into account. Besides producing electricity, a unit can also be in "reserve", which means that some of its power is put aside to provide electricity, in case of a shortcoming somewhere over the network. A revenue is earned through this practice depending on the type of reserve under consideration, which can be either "10 minutes spin", "10 minutes non spin" or "30 minutes non spin". It should be noted that when all the power of a unit is assigned to the "10 minutes spin" reserve, a cost is incurred because this unit needs some electricity to spin at the right speed to synchronize itself with the electric network.

1.2 Electricity market model

In our application, the electricity prices are set in a deregulated market, where producers and buyers submit selling and purchase bids, respectively. Every bid corresponds to an amount of energy and a selling or purchase price per MWh. Supply and demand curves are then created and the intersection of the two curves determines the Market Clearing Price (MCP). All selling bids under the MCP and all purchase bids over the MCP are accepted. A producer with at least one accepted bid will receive the MCP for the production of each MWh of energy. The buyer with at least one accepted bid will pay the MCP for the use of each MWh of energy.

There are also three other markets associated with the three different reserve types. However, the "30 minutes non spin" reserve is not considered here because its selling price is always less than or equal to the "10 minutes non spin" reserve price, and there is no cost for providing these two reserves. The market price for each reserve type is calculated by considering the selling bids from producers and the amount of reserve required to get a safe electric network.

In this study, the market clearing prices are based on distributions that follow the historic evolution of prices over a typical day. The exact stochastic processes for generating these prices are formally introduced in a later section.

1.3 Objective

The goal is to find a bidding strategy, for each unit in the power plants along the river, that maximizes the profit resulting from electricity and reserve selling, while satisfying specific operational, physical and financial constraints. The result is a table indicating, for each hour and each unit, the fraction of the capacity associated with electricity production and the two types of reserve, with their respective selling prices, over a 24-hour horizon.

To avoid emptying reservoirs, the profit value is modified through the use of target volumes of water. That is, when the volume of water in a reservoir is above the target at the end of the planning horizon, a reward or bonus is added according to the "value" of this water (based on long-term price averages). Conversely, a penalty is incurred if the final volume is below the target.

2 Related work

The literature related to our problem can be divided into two classes. The first class contains papers that focus on water management at physical facilities under various constraints. In these papers, the facilities are modeled in a realistic way, but market considerations are either absent or very simplistic.

The second class contains papers dealing with bidding strategies in deregulated electricity markets. Some consider "price-taker" market participants, while others consider "price-searcher" participants. Price-takers are producers whose bids do not influence the MCPs. They are basically small players. On the other hand, price searchers can influence the clearing prices through strategic bidding. Most papers consider the viewpoint of a single producer in a market with no competition among the participants. It means that the producer under study develops a bidding strategy while the others bid as usual. A few papers integrate competition among the producers to obtain a more realistic model.

Papers that combine water management issues with strategic bidding are uncommon. Indeed, we found only one paper that considers strategic bidding for a series of power plants along a river (see section 2.2.1).

2.1 Water management

Water management in the hydroelectricity field has been studied intensively over the last decades and different techniques for solving single or multiple reservoir problems have been developed over the years. Multiple reservoir problems are much more difficult to solve due to the "cascade effects" that take place from one reservoir to the next. Multiple reservoir problems have been tackled in many different settings:

- with deterministic natural inflows and no market price considerations [2, 21, 28, 32, 33, 36, 44];
- with stochastic natural inflows and no market price considerations [1, 3, 6, 13, 29];
- with deterministic natural inflows and market price considerations [9, 15, 17, 18, 30, 37];

- with stochastic natural inflows and market price considerations [14].

Also, a wide variety of techniques have been proposed to solve these problems, like:

- network programming [2, 14, 18];
- linear programming [14, 17, 28, 32, 36, 37];
- mixed integer programming [9, 15, 36, 37];
- dynamic programming [1, 3, 6, 13, 14, 18, 29, 30];
- nested Benders decomposition [3];
- lagrangian relaxation [18, 33];
- branch-and-bound [2];
- specialized heuristics and metaheuristics [21, 33, 44].

2.2 Strategic bidding in deregulated markets

Deregulated electricity markets are relatively new. Research activities have mostly focused on ways to improve competitiveness in these markets rather than maximizing profits for energy production companies. Indeed, all papers reported in the preceding section, except for one, do not consider the bidding process. This is definitely a weakness, because a clever bidding strategy can take advantage of the price volatility observed in deregulated markets to sell electricity when prices are high. In a hydroelectricity production context, this is even more important because of the limited amount of water available.

2.2.1 Price-takers

In a perfect competitive market, many small producers are involved. In this context, it is not possible for a producer to influence the MCPs. A price-taker is thus a producer who must accept the market prices without having the possibility to influence them.

In [10], the authors propose a mixed integer programming model to optimize the production schedule of a single thermal unit. A simple strategy is introduced to convert the production schedule into real bids. The strategy consists in bidding very low when the unit must produce electricity and very high, otherwise. Although this approach works well in the presence of good price forecasts, additional benefits can be obtained in a volatile price environment by considering more flexible bidding strategies.

In [23], a pumped-storage unit is considered. This kind of hydro-turbine is well adapted to deregulated markets due to its inherent flexibility. In particular, it is possible to produce electricity during peek hours and to pump water back in the reservoir during periods associated with low prices. Bids are not explicitly considered in the optimization model, that is, only a production schedule is generated. The problem is solved in two phases by an iterative algorithm. In the first phase, the problem is solved without considering constraints and in the second phase, a special procedure is used to find and repair violated constraints. In [38], the authors present a mixed integer stochastic optimization model for the scheduling of thermal units. In this model, production plans are optimized in the presence of stochastic MCPs, without considering bids explicitly. Thus, an ad hoc bidding strategy, like the one mentioned above, must be used.

An electricity market based on a linear supply bid function is considered in [34]. In this type of market, the producers must submit the two coefficients of their linear supply function. The latter returns the amount of electricity that they are willing to produce for a given selling price. The authors develop a stochastic programming model for this problem, which is solved with MATLAB after a transformation into a non-linear deterministic model. A methodology to build coordinated bidding strategies is also proposed in the same context in [41]. The participants choose their bid coefficients to maximize their profits based on their knowledge of the other participants' behavior (i.e., how the other participants will react to their bids). The authors develop a stochastic model which is solved with a combined Monte Carlo and genetic algorithm approach.

In [26], the authors consider thermal and pumped-storage plants with stochastic MCPs in a day-ahead market. Bids are optimized using stochastic mixed integer programming. When the bids are known, a lagrangian relaxation and a stochastic dynamic programming algorithm are used to solve the resulting

unit commitment problem. A lagrangian relaxation is also proposed in [16] to find an optimal bidding strategy for a thermal unit. In [11, 19], bids are optimized for price-taker producers, while taking into account competition among producers through a Nash equilibrium model.

The work of Pritchard and Zakeri in [30] is the most similar to ours and is the only one that integrates the bidding process into a hydrologic system with multiple reservoirs. The authors develop a stochastic dynamic programming (SDP) model to maximize the profits resulting from electricity selling in a deregulated market, where electricity prices are modeled through non homogeneous Markov chains. To solve their problem with SDP, the reservoir levels are first discretized. Then, the river routing effects are taken into account by introducing artificial intermediate reservoirs for each time period that falls in the gap between the water release at an upstream reservoir and the water arrival at the next reservoir. The size of the state space grows quickly with the number of reservoirs considered (including artificial ones). As a consequence, only small problems with two reservoirs, plus two intermediate ones, are addressed in their work. Another shortcoming is that their approach is quite restricted with regard to the type of price generation processes that can be handled. Also, some practical issues are not addressed (but could be integrated at the expense of an increase in complexity), like start-up costs and reserve selling.

The problem-solving approach proposed by Pritchard and Zakeri is certainly valuable, but is not appropriate in our context because nine intermediate reservoirs would have to be introduced in addition to the four real reservoirs (i.e., three reservoirs between each consecutive pair of reservoirs). With only ten discretization levels per reservoir, a state space with more than 10^{13} states per period would be obtained.

The alternative model proposed in this paper grows linearly with the number of reservoirs, through the management of a tree of price scenarios (see section 3). A shortcoming of this approach is that it prevents us from considering mid- or long-term time horizons. However, this is not required in our application because only the next 24 hours need to be considered. We also have flexibility with regard to the stochastic process used for generating electricity and reserve prices. This would allow our system to be fed, for example, with prices obtained from an external forecast module (see section 2.3).

2.2.2 Price-searchers

Price-searchers are participants with "market power", that is, their bidding decisions have an impact on the MCPs. Most of the time, price-searchers are big companies in a market with only a small number of competitors. These kinds of markets are known as oligopoly. The work reported in [7, 27, 35, 43, 45] describes bid optimization models for this type of market.

A technique to handle price-searcher participants in a market where the bids are based on linear supply curves with coefficients is described in [25, 40, 42]. The authors consider a limited number of producers with only one main player (the one whose profit is to be optimized). The idea is to model the distribution of the coefficients in the supply curves of the competitors to optimize the main player's coefficients, by considering a simplified market model.

Another way to handle price-searchers is to use residual demand curves. The idea is to forget about the decisions of the other players and to focus only on the consequences of the main player's decisions on the MCPs. More precisely, a curve for the MCPs versus the quantities offered is created [5, 31]. This approach has the advantage to simplify the model and to make the problem tractable for markets with many players. Furthermore, it eliminates the need to evaluate coefficients in the supply function of each participant.

Competition arises in deregulated electricity markets when participants develop bid strategies aimed at guessing what other participants will do. In perfect competitive markets, the participants have an incentive to bid at their marginal cost but, in oligopoly contexts, competition and "market power" are typically observed. The work reported in [4, 8, 39] are developed in this context. In [4], for example, game theory is used in conjunction with a Nash-Cournot equilibrium model to integrate competition issues.

2.3 Price forecasts

In deregulated markets, the ability to predict electricity and reserve prices with accuracy is crucial in the elaboration of good bidding strategies. This is quite complex, however, because many unpredictable events can influence the prices. For example, an important transmission line can be damaged or a plant can unexpectedly shut down. On the other hand, predictable factors, like weather forecasts and consumption habits, can provide a good starting point. In [20], the authors describe a neuro-fuzzy approach to forecast electricity prices under normal conditions (i.e., without unexpected events). In [24],

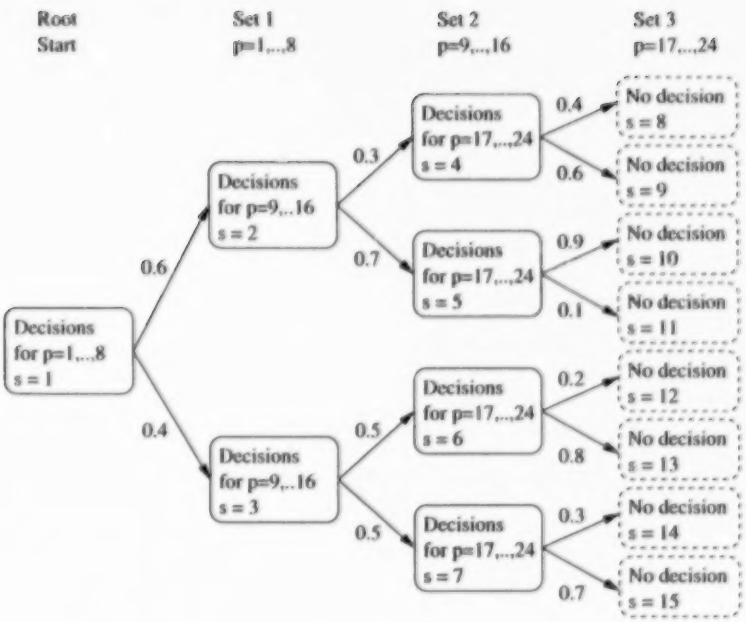

Figure 2: Tree of scenarios

data mining techniques are used to forecast price "spikes". Finally, a comparison of price volatility in different deregulated electricity markets is provided in [22].

3 Mathematical model

The model presented here is developed for a price-taker participant. That is, the bids of the producer have no impact on the MCPs. Furthermore, the model does not consider competition among participants. In the model, different scenarios or realizations for the MCPs are considered. The scenarios are organized into a tree structure where each node, at a given level, contains a particular realization of MCPs for the set of periods associated with that level. From each node, the children nodes at the next level can be reached with a certain (conditional) probability. The root node corresponds to the "empty scenario" at the start of the production horizon, where no prices have yet realized. Figure 2 shows a tree of scenarios over an horizon of 24 periods, where the scenarios at the first level are associated with the set of periods {1,...,8}, those at the second level with set {9,...,16} and those at the third level with set {17,...,24}.

At each level, including the root but excluding the leaves, selling bids are made for the set of periods at the next level, without knowing with certainty the electricity and reserve MCPs for these periods. In our example, bids are first taken at the root for periods 1 to 8, without knowing exactly the MCPs for these periods. Then, based on the particular realization observed for these prices, another node is reached at the next level and new bids are made for periods 9 to 16, etc. Although the prices are not known exactly when the bids are taken, the conditional probability of observing a particular realization of prices over the next set of periods, given the prices observed up to now, is assumed to be known.

The overall expected profit is then the summation over all internal nodes in the tree of the probability of being in that node (as obtained by multiplying the conditional probabilities on the path leading from the root to the node) times the conditional expected profit from that node over the next set of periods. For example, the probability of being in the node associated with scenario $s = 7$ is equal to $0.4 * 0.5 = 0.2$. Note that the leaves are left apart, given that no decisions are taken there.

The shape of the tree depends on the sets of periods that are associated with each level. For example, a much deeper tree would be obtained by associating a single period with each level. Also, the branching factor in the tree corresponds to the number of different realizations of MCPs that are considered for

each set of periods. Thus, the following parameters determine the shape of the scenario tree:

- N_b: number of branches at each level (branching factor);
- N_s: number of sets of periods (number of levels);
- p_j: last period in period set j, $j = 1, ..., N_s$.

To associate MCPs with the nodes in the tree of scenarios, we first generate N_v vectors of prices of cardinality P, where P is the number of periods, based on some underlying stochastic process (the one used in our experiments is formally introduced in Section 4). Given a branching factor N_b and a partition of the P periods into N_s sets of periods, we sort these vectors based on the price of period p_1 (last period of the first set), starting with the smallest price. We then take price averages for periods 1 to p_1 based on the first $\lfloor N_v/N_b \rfloor$ vectors, which are those with the smallest prices, and we associate these average MCPs and the corresponding subset of price vectors with the first child node.

We repeat for the next child node by averaging over the next $\lfloor N_v/N_b \rfloor$ vectors. This is repeated until every child node of the root is done. Then, we go to the next level and repeat the whole procedure again for every node at that level: we sort the subset of vectors associated with the current node based on the price of period p_2 (last period of the second set of periods) and associate a (sub)subset of vectors and the corresponding average MCPs with every child. The procedure stops at the leaves of the scenario tree.

At each internal node, the decision variables are the bids for the prices and quantities of electricity to be produced over the next set of periods. The production variables are derived from these decision variables. For the "10 minutes spin reserve" and "10 minutes non spin reserve", it is assumed that the bids are set at the minimum price. Consequently, these bids are always accepted by the market and the only decision in this case relate to the quantities of electricity to be reserved. Basically, once a decision is taken about the quantities to be put aside for the two reserves, it is always advantageous to "sell" them (as the reserve is rarely asked for in practice; thus no water is consumed and the associated costs are low).

The mathematical model will now be formally introduced. It is based on the tree of scenarios previously mentioned, where each node is associated with a set of periods, a MCP scenario (realization of market clearing prices up to this set of periods) and bid decisions for the next set of periods. These bid decisions are modeled with binary variables. Basically, a number of bid choices are available and a variable is associated with each possible choice. To simplify the formulation, the model does not handle sets of periods explicitly, but rather handles each period individually.

3.1 Parameters

- P : number of periods.
- D : number of dams.
- U_d : number of units in dam d, $d = 1, ..., D$.
- S: number of nodes in the scenario tree.
- S^{int}: number of internal nodes in the scenario tree (nodes where a decision is taken).
- S_p : set of nodes associated with period p, $p = 1, ..., P$.
- S_p^{int} : set of internal nodes where a decision is taken for period p, $p = 1, ..., P$.
- $PARENT_s$: parent of node s in the scenario tree, $s = 2, ..., S$.
- $PARENT_{s,p,t}$: ancestor of node s associated with period $p-t$, $p = 1, ..., P$; $s \in S_p$; $t = 0, ..., p-1$. Note that the ancestor node can be the same as node s if p and $p-t$ are in the same set of periods.
- $CHILDREN_s$: set of children of node s in the scenario tree, $s = 1, ..., S^{int}$.
- $PROB_{s,s_c}$: conditional probability to go from node s to its child node s_c in the tree of scenarios, $s = 1, ..., S$; $s_c \in CHILDREN_s$.
- $PROB_{p,s}$: probability to be in node s at period p. It is obtained by multiplying the conditional probabilities on the path from the root to the corresponding scenario in the tree of scenarios, $p = 1, ..., P$; $s \in S_p$. These probabilities are the same for all periods in the same set of periods.
- $P_{p,s}^{ENGY}$: MCP for each MWh of energy produced at period p in node s ($/MWh), $p = 1, ..., P$; $s \in S_p$.
- $P_{p,s}^{10S}$: MCP for each MWh in "10 minutes spin reserve" at period p in node s ($/MWh), $p = 1, ..., P$; $s \in S_p$.
- $P_{p,s}^{10N}$: MCP for each MWh in "10 minutes non spin reserve" at period p in node s ($/MWh), $p = 1, ..., P$; $s \in S_p$.

- DUR_p : duration of period p in hours, $p = 1, ..., P$.
- $PROD_{d,u}(f)$: power of unit u of dam d as a function of flow f (MW), $d = 1, ..., D$; $u = 1, ..., U_d$ (see Section 3.7).
- $I_{d,u}$: number of piecewise parts in the linear approximation of $PROD_{d,u}(f)$, $d = 1, ..., D$; $u = 1, ..., U_d$. (see Section 3.7).
- $RATE_{d,u,i}$: rate (slope) of part i in the linear approximation of $PROD_{d,u}(f)$, $d = 1, ..., D$; $u = 1, ..., U_d$; $i = 1, ..., I_{d,u}$ (see Section 3.7).
- $LIM_{d,u,i}$: upper bound or limit of part i in the linear approximation of $PROD_{d,u}(f)$, $d = 1, ..., D$; $u = 1, ..., U_d$; $i = 1, ..., I_{d,u} - 1$ (see Section 3.7).
- $VOL_{d,p}^{min}$: minimum volume of dam d at the end of period p (m^3), $d = 1, ..., D$; $p = 1, ..., P$.
- $VOL_{d,p}^{max}$: maximum volume of dam d at the end of period p (m^3), $d = 1, ..., D$; $p = 1, ..., P$.
- VOL_d^{init} : initial volume of dam d (at t. e end of period 0) (m^3), $d = 1, ..., D$.
- VOL_d^{final} : target volume of dam d at the end of the last period P (m^3/s), $d = 1, ..., D$.
- $F_{d,u}^{min}$: minimum amount of flow that can be processed by unit u of dam d (m^3/s), $d = 1, ..., D$; $u = 1, ..., U_d$.
- $F_{d,u}^{max}$: maximum amount of flow that can be processed by unit u of dam d (m^3/s), $d = 1, ..., D$; $u = 1, ..., U_d$.
- $F_d^{spill_{min}}$: minimum spill flow of dam d (m^3/s), $d = 1, ..., D$.
- $F_d^{spill_{max}}$: maximum spill flow of dam d (m^3/s), $d = 1, ..., D$.
- $F_d^{out_{min}}$: minimum outflow of dam d (m^3/s), $d = 1, ..., D$.
- $F_d^{out_{max}}$: maximum outflow of dam d (m^3/s), $d = 1, ..., D$.
- $MW_{d,u}^{max}$: maximum power of unit u of dam d (MW), $d = 1, ..., D$; $u = 1, ..., U_d$.
- NAT_IFd, p : mean natural inflow forecast at dam d for period p (m^3/s), $d = 1, ..., D$; $p = 1, ..., P$.
- $LK_{z,d}$: 1 if dam z is the immediate predecessor of dam d, 0 otherwise, $z = 1, ..., D$; $d = 1, ..., D$.
- N : number of periods that are considered when calculating the river routing effect.
- $LK_FRAC_{z,d,t}$: fraction ($\in [0, 1]$) of water, released in period $p - t$ at dam z, that arrives at dam d in period p (same value for all p), $z = 1, ..., D$; $d = 1, ..., D$; $t = 0, ..., N$.
- $MW_{d,u}^{spin}$: power needed by unit u of dam d to make it spin (MW), $d = 1, ..., D$; $u = 1, ..., U_d$.
- $C_Start_{d,u}$: cost for each start up of unit u of dam d ($), $d = 1, ..., D$; $u = 1, ..., U_d$.
- E_d : estimate of the energy that can be produced for each m^3 of water at dam d (MWh/m^3), $d = 1, ..., D$. This value gets larger as we move upstream (upward) because more electricity can be produced with the same amount of water, due to the cascade effect, $d = 1, ..., D$.
- LT^{ENGY} : estimate of the average long-term electricity prices ($/MWh).
- $N_{p,s}^{ENGY}$: number of different bid choices for period p at node s, $p = 1, ..., P$; $s \in S_p^{int}$.
- $BID_{p,s,b}^{ENGY}$: price associated with bid choice b for period p at node s, $p = 1, ..., P$; $s \in S_p^{int}$; $b = 1, ..., N_{p,s}^{ENGY}$.
- $C_{p,s,b}^{ENGY}$: 1 if $BID_{p,PARENT_s,b}^{ENGY} \leq P_{p,s}^{ENGY}$ (bid is accepted), 0 otherwise, $p = 1, ..., P$; $s \in S_p$; $b = 1, ..., N_{p,PARENT_s}^{ENGY}$.

3.2 Decision variables

- $bid_{d,u,p,s,b}^{ENGY} \in \{0, 1\}$: 1 if bid choice b is chosen for unit u of dam d when a decision for period p is taken in node s, 0 otherwise, $d = 1, ..., D$; $u = 1, ..., U_d$; $p = 1, ..., P$; $s \in S_p^{int}$; $b = 1...N_{p,s}^{ENGY}$.
- $bid_mw_{d,u,p,s}^{ENGY} \geq 0$: bid on quantity to be produced by unit u of dam d when a decision for period p is taken in node s, $d = 1, ..., D$; $u = 1, ..., U_d$; $p = 1, ..., P$; $s \in S_p^{int}$.
- $bid_mw_{d,u,p,s}^{10S} \geq 0$: bid on quantity for "10 minutes spin reserve" by unit u of dam d when a decision for period p is taken in node s, $d = 1, ..., D$; $u = 1, ..., U_d$; $p = 1, ..., P$; $s \in S_p^{int}$.
- $bid_mw_{d,u,p,s}^{10N} \geq 0$: bid on quantity for "10 minutes non spin reserve" by unit u of dam d when a decision for period p is taken in node s, $d = 1, ..., D$; $u = 1, ..., U_d$; $p = 1, ..., P$; $s \in S_p^{int}$.
- $f_{d,p,s}^{spill} \geq 0$: flow spilled at dam d at period p in node s (m^3/s), $d = 1, ..., D$; $p = 1, ..., P$; $s \in S_p$. Note that the spill decisions are derived once the accepted bids are known.

3.3 Constraints on decision variables

- (C1) Exactly one bid choice is selected for period p at node s for unit u of dam d, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p^{int}$.

$$\sum_{b=1...N_{p,s}^{ENGY}} bid_{d,u,p,s}^{ENGY} = 1$$

- (C2) Maximum power of unit u of dam d is used in the bids of period p at node s, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p^{int}$.

$$bid_mw_{d,u,p,s}^{ENGY} + bid_mw_{d,u,p,s}^{10S} + bid_mw_{d,u,p,s}^{10N} = MW_{d,u}^{max}$$

- (C3) Flow spilled at dam d in period p is between a minimum and a maximum value, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$F_d^{spill_{min}} \leq f_{d,p,s}^{spill} \leq F_d^{spill_{max}}$$

3.4 Intermediate variables

Here is a list of intermediate variables. The values of these variables depend on the parameters and the original decision variables. They are used here to simplify the formulation of the constraints that follow.

- $b_{d,u,p,s}^{ENGY} \in \{0, 1\}$: 1 if electricity is produced by unit u of dam d in period p at node s, 0 otherwise, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$b_{d,u,p,s}^{ENGY} = \sum_{b=1...N_{p,s}^{ENGY}} C_{p,s,b}^{ENGY} \cdot bid_{d,u,p,PARENT_s,b}^{ENGY}$$

- $f_{d,u,p,s}^{ENGY} \geq 0$: flow going through unit u of dam d to produce electricity in period p at node s (m^3/s), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.
- $f_{d,u,p,i,s}^{ENGY} \geq 0$: contribution of piecewise part i in the linear approximation of $PROD_{d,u}(f)$ to the flow going through unit u of dam d in period p at node s to produce electricity (m^3/s), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ i = 1, ..., I_{d,u};\ s \in S_p$ (see Section 3.7).
- $b_{d,u,p,s}^{spin} \in \{0, 1\}$: 1 if unit u of dam d spins by supplying electricity to it in period p at node s, 0 otherwise, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.
- $mw_{d,u,p,s}^{ENGY} \geq 0$: power used by unit u of dam d in period p at node s to produce electricity (MW), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.
- $mw_{d,u,p,s}^{10S} \geq 0$: power reserved for "10 minutes spin reserve" for unit u of dam d in period p at node s (MW), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$mw_{d,u,p,s}^{10S} = bid_mw_{d,u,p,PARENT_s}^{10S}$$

- $mw_{d,u,p,s}^{10N} \geq 0$: power reserved for "10 minutes non spin reserve" for unit u of dam d in period p at node s (MW), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$mw_{d,u,p,s}^{10N} = bid_mw_{d,u,p,PARENT_s}^{10N}$$

- $outflow_{d,p,s}$: flow that leaves the reservoir of dam d in period p at node s (m^3/s), $d = 1, ..., D;\ p = 1, ..., P;\ s \in S_p$. It is also assumed that a number of "historical" outflow values are available for periods $-(N-1), ..., 0$.
- $inflow_{d,p,s}$: flow that enters the reservoir of dam d in period p at node s (m^3/s), $d = 1, ..., D;\ p = 1, ..., P;\ s \in S_p$.

$$inflow_{d,p,s} = NAT_IF_{d,p} + \sum_{\substack{z=1,...,D \\ t=0,...,N}} LK_{z,d} \cdot outflow_{z,p-t,PARENT_{s,p,t}} \cdot LK_FRAC(z,d,t)$$

- $outflow_{d,p,s}$: flow that leaves the reservoir of dam d in period p at node s (m^3/s), $d = 1, ..., D;\ p = 1, ..., P;\ s \in S_p$. It is also assumed that a number of "historical" outflow values are available for periods $-(N-1), ..., 0$.

$$outflow_{d,p,s} = f_{d,p,s}^{spill} + \sum_{u=1,...,U_d} f_{d,u,p,s}^{ENGY}$$

- $vol_{d,p,s}$: volume of dam d at the end of period p at node s (m^3), $d = 1, ..., D;\ p = 1, ..., P;\ s \in S_p$.

$$p = 1: \quad vol_{d,p,s} = VOL_d^{init} + (inflow_{d,p,s} - outflow_{d,p,s}) \cdot DUR_p \cdot 3600$$
$$p = 2, ..., P: \quad vol_{d,p,s} = vol_{d,p-1,Parent_{s,p,1}} + (inflow_{d,p,s} - outflow_{d,p,s}) \cdot DUR_p \cdot 3600$$

3.5 Constraints on intermediate variables

- (C4) $f^{ENGY}_{d,u,p,s}$ is the sum of all piecewise parts, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$f^{ENGY}_{d,u,p,s} = \sum_{i=1,...,I_{d,u}} f^{ENGY}_{d,u,p,i,s}$$

- (C5) Bounds on $f^{ENGY}_{d,u,p,s}$, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$0 \leq f^{ENGY}_{d,u,p,s} \leq F^{max}_{d,u}$$

- (C6) Bounds on $f^{ENGY}_{d,u,p,i,s}$, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ i = 1, ..., I_{d,u};\ s \in S_p$.

$$\begin{aligned} i = 1: &\quad 0 \leq f^{ENGY}_{d,u,p,i,s} \leq LIM_{d,u,1} \\ i = 2, ..., I_{d,u} - 1: &\quad 0 \leq f^{ENGY}_{d,u,p,i,s} \leq LIM_{d,u,i+1} - LIM_{d,u,i} \\ i = I_{d,u}: &\quad 0 \leq f^{ENGY}_{d,u,p,i,s} \leq F^{max}_{d,u} \end{aligned}$$

- (C7) Calculation of $mw^{ENGY}_{d,u,p,s} = PROD_{d,u}(f^{ENGY}_{d,u,p,s})$ through the linear approximation, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$mw^{ENGY}_{d,u,p,s} = \sum_{i=1,...,I_{d,u}} (f^{ENGY}_{d,u,p,i,s} \cdot RATE_{d,u,i}) - b^{ENGY}_{d,u,p,s} \cdot F^{min}_{d,u} \cdot RATE_{d,u,1}$$

- (C8) $f^{ENGY}_{d,u,p,s}$ equals 0 if $b^{ENGY}_{d,u,p,s}$ is 0 and is between a minimum and a maximum value otherwise, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$F^{min}_{d,u} \cdot b^{ENGY}_{d,u,p,s} \leq f^{ENGY}_{d,u,p,s} \leq F^{max}_{d,u} \cdot b^{ENGY}_{d,u,p,s}$$

- (C9) Power produced equals 0 if $b^{ENGY}_{d,u,p,s} = 0$ and equals $bid_mw^{ENGY}_{d,u,p,PARENT_s}$ otherwise, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$\begin{aligned} mw^{ENGY}_{d,u,p,s} &\leq bid_mw^{ENGY}_{d,u,p,PARENT_s} \\ mw^{ENGY}_{d,u,p,s} &\leq b^{ENGY}_{d,u,p,s} \cdot MW^{max}_{d,u} \end{aligned}$$

- (C10) Power for "10 minutes spin reserve" can only be reserved if unit u of dam d spins or produces electricity, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$mw^{10S}_{d,u,p,s} \leq MW^{max}_{d,u} \cdot (b^{ENGY}_{d,u,p,s} + b^{spin}_{d,u,p,s})$$

- (C11) Unit u of dam d cannot produce electricity and spin at the same time, $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$b^{ENGY}_{d,u,p,s} + b^{spin}_{d,u,p,s} \leq 1$$

- (C12) Volume of dam d at the end of period p in node s is between a minimum and maximum value, $d = 1, ..., D;\ p = 1, ..., P;\ s \in S_p$.

$$VOL^{min}_{d} \leq vol_{d,p,s} \leq VOL^{max}_{d}$$

- (C13) Outflow of dam d in period p in node s is between a minimum and maximum value, $d = 1, ..., D;\ p = 1, ..., P;\ s \in S_p$.

$$F^{out_{min}}_{d} \leq outflow_{d,p,s} \leq F^{out_{max}}_{d}$$

3.6 Objective

The objective is to maximize the expected profit, which relates to the expected MCPs (as the latter determine if the bids are accepted or not). In the formula below, this is hidden in variables $revenue_{d,u,p,s_c}$ that depend on variables $mw^{ENGY}_{d,u,p,s}$. The latter correspond to the power used to produce electricity and depend on bids through constraints (C9).

- Objective function (\$)

$$\begin{aligned} Maximize \sum_{\substack{d\in\{1,...,D\} \\ u\in\{1,...,U_d\} \\ p\in\{1,...,P\} \\ s\in S^{int}_p}} Prob_{p,s} \sum_{s_c \in CHILDREN_s} Prob_{s,s_c}(revenue_{d,u,p,s_c} - c^{spin}_{d,u,p,s_c} - c^{start}_{d,u,p,s_c}) + \\ \sum_{\substack{d\in\{1,...,D\} \\ s\in S^{int}_p}} Prob_{P,s} \sum_{s_c \in CHILDREN_s} Prob_{s,s_c} \cdot volAdj_{d,s_c} \end{aligned}$$

This objective can be simplified as follows:

$$Maximize \sum_{\substack{d\in\{1,...,D\}\\ u\in\{1,...,U_d\}\\ p\in\{1,...,P\}\\ s\in S_p}} Prob_{p,s}(revenue_{d,u,p,s} - c^{spin}_{d,u,p,s} - c^{start}_{d,u,p,s}) + \sum_{\substack{d\in\{1,...,D\}\\ s\in S_P}} Prob_{P,s} \cdot volAdj_{d,s}$$

- $revenue_{d,u,p,s}$: revenue from selling electricity and reserve for unit u of dam d in period p at node s ($), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$revenue_{d,u,p,s} = (mw^{ENGY}_{d,u,p,s} \cdot P^{ENGY}_{p,s} + mw^{10S}_{d,u,p,s} \cdot P^{10S}_{p,s} + mw^{10N}_{d,u,p,s} \cdot P^{10N}_{p,s}) \cdot DUR_p$$

- $c^{spin}_{d,u,p,s}$: cost for providing electricity to unit u of dam d to make it spin in period p at node s ($), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$c^{spin}_{d,u,p,s} = b^{spin}_{d,u,p,s} \cdot MW^{spin}_{d,u} \cdot P^{ENGY}_{p,s} \cdot DUR_p$$

- $c^{start}_{d,u,p,s}$: start up cost of unit u of dam d in period p at node s ($), $d = 1, ..., D;\ u = 1, ..., U_d;\ p = 1, ..., P;\ s \in S_p$.

$$p = 1: \quad c^{start}_{d,u,p,s} \geq C_START_{d,u} \cdot b^{ENGY}_{d,u,p,s},$$
$$p = 2, ..., P: \quad c^{start}_{d,u,p,s} \geq C_START_{d,u} \cdot (b^{ENGY}_{d,u,p,s} - b^{ENGY}_{d,u,p-1,PARENT_{s,p,1}}),$$
$$c^{start}_{d,u,p,s} \geq 0$$

- $volAdj_{d,s}$: value of water at dam d at the end of planning horizon at node s ($), $d = 1, ..., D;\ s \in S_P$.

$$volAdj_{d,s} = (vol_{d,P,s} - VOL^{final}_d) \cdot E_d \cdot LT^{ENGY}$$

3.7 Power generation function

For a specific unit, the power generation function $PROD_{d,u}$ is typically not linear, since the marginal increase in power tends to diminish with the flow increase. We thus approximate it through a concave piecewise linear function. An example is presented in Figure 3, using a fixed reference value for the water head. We note that there is no power until a flow that corresponds to $F^{min}_{d,u}$ is reached. The general idea is to divide the function into $I_{d,u}$ intervals with bounds $LIM_{d,u,i}$, $i = 1, ..., I_{d,u} - 1$ and $F^{max}_{d,u}$. With each interval is also associated a slope $RATE_{d,u,i}$, $i = 1, ..., I_{d,u}$. We then define $I_{d,u}$ variables $f^{ENGY}_{d,u,p,i,s}$, $i = 1, ..., I_{d,u}$, and we impose that $f^{ENGY}_{d,u,p,s} = \sum_{i=1,...,I_{d,u}} f^{ENGY}_{d,u,p,i,s}$. Note that the concavity of the function is required for this model to work.

The value of $PROD_{d,u}(f^{ENGY}_{d,u,p,s})$ is then approximated with the following formula:

$$\sum_{i=1,...,I_{d,u}} (f^{ENGY}_{d,u,p,i,s} \cdot RATE_{d,u,i}) - b^{ENGY}_{d,u,p,s} \cdot F^{min}_{d,u} \cdot RATE_{d,u,1}.$$

That is, we sum the flow value associated with each interval times the rate and, if some energy is produced ($b^{ENGY}_{d,u,p,s} = 1$), we subtract $F^{min}_{d,u} \cdot RATE_{d,u,1}$ because the function does not start at 0 but at $F^{min}_{d,u}$.

4 Numerical results

This section presents the results of different experiments with the model previously described that integrates a bidding strategy. It is compared with a deterministic and a stochastic model already reported in a previous paper [12]. The three models are based on the same stochastic process for generating the MCPs (see below). In the deterministic model, the MCPs are assumed to be known with certainty and are set at the average price of the distribution. A production plan is then generated accordingly. The stochastic model in [12] is closely related to the one that integrates bids, in the sense that the same tree of scenarios is used to take into account various realizations of the MCPs. The decisions, however, relate only to the quantities of electricity to be produced versus the quantities to be allowed for the reserves (which are then automatically sold at the MCPs, since there are no bidding prices to be compared with the MCPs).

We first examine the impact of the production horizon on the three models. Then, we focus on the two stochastic models, given that the superiority of the original stochastic model (without bids) over the deterministic model was already demonstrated in [12]. We first compare the two stochastic models based on different values for the branching factor and the number of period sets. Then, we examine the impact of volatility and "spikes" in the price patterns.

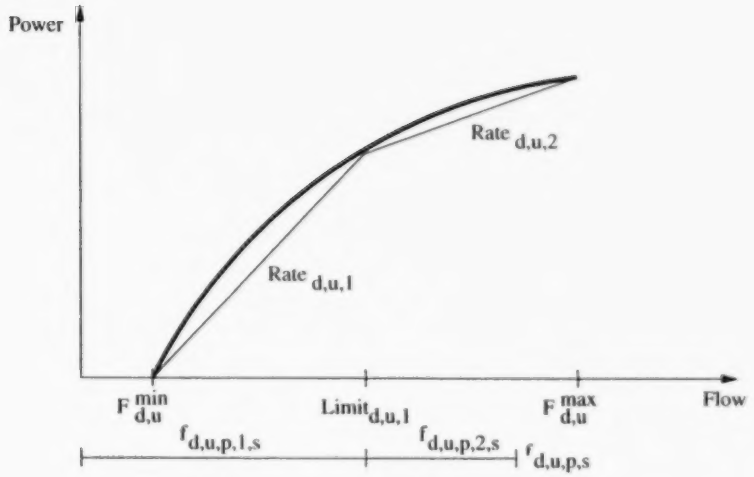

Figure 3: Example of a power generation function

4.1 Creating stochastic models

The two stochastic models and the deterministic model exploit average electricity ($\overline{P}_p^{ENGY}$) and reserve prices ($\overline{P}_p^{10S}$, $\overline{P}_p^{10N}$) at each period $p \in P$. In the two stochastic models, the branching factor N_b at each level in the tree of scenarios and the number of sets of periods or levels N_s are first specified. Then, the value of the p_j's are chosen to get (as much as possible) the same number of periods at each level. That is:

$$p_j = \lfloor P \cdot j/N_s \rfloor, \quad j = 1, ..., N_s - 1,$$
$$p_j = P, \quad j = N_s.$$

The stochastic process for generating electricity prices that are used in the tree of scenarios, denoted ϕ_p^{ENGY}, $p = 1, ..., P$, is based on trends observed in real data and can be described as follows:

- $p = 1$,

$$\phi_p^{ENGY} = \begin{cases} \pi_p \cdot a_1 & if\ \epsilon_p \leq Q, \\ \pi_p \cdot (2 - a_1) & if\ Q < \epsilon_p \leq 2Q, \\ \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = 2, ..., \lfloor P/4 \rfloor$,

$$\phi_p^{ENGY} = \begin{cases} \phi_{p-1}^{ENGY} \cdot \pi_p \cdot a_1 & if\ \epsilon_p \leq Q, \\ \phi_{p-1}^{ENGY} \cdot \pi_p \cdot (2 - a_1) & if\ Q < \epsilon_p \leq 2Q, \\ \phi_{p-1}^{ENGY} \cdot \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = \lfloor P/4 \rfloor + 1, ..., \lfloor 2P/4 \rfloor$,

$$\phi_p^{ENGY} = \begin{cases} \phi_{p-1}^{ENGY} \cdot \pi_p \cdot a_2 & if\ \epsilon_p \leq Q, \\ \phi_{p-1}^{ENGY} \cdot \pi_p \cdot (2 - a_2) & if\ Q < \epsilon_p \leq 2Q, \\ \phi_{p-1}^{ENGY} \cdot \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = \lfloor 2P/4 \rfloor + 1, ..., \lfloor 3P/4 \rfloor$,

$$\phi_p^{ENGY} = \begin{cases} \phi_{p-1}^{ENGY} \cdot \pi_p \cdot a_3 & if\ \epsilon_p \leq Q, \\ \phi_{p-1}^{ENGY} \cdot \pi_p \cdot (2 - a_3) & if\ Q < \epsilon_p \leq 2Q, \\ \phi_{p-1}^{ENGY} \cdot \pi_p & if\ \epsilon_p > 2Q \end{cases}$$

- $p = \lfloor 3P/4 \rfloor + 1, ..., P$,

$$\phi_p^{ENGY} = \phi_{p-1}^{ENGY} \cdot \pi_p + \frac{\overline{P}_p^{ENGY} - \phi_{p-1}^{ENGY} \cdot \pi_p}{1+\alpha},$$

where a_1, a_2 and a_3 are the "volatility" parameters, α is a parameter that controls the convergence rate toward the average, $2Q$ is the probability to deviate from the average price (probability Q to be over the average; probability Q to be under the average), ϵ_p, $p = 1, ..., \lfloor 3P/4 \rfloor$, are independent uniform random variables in the interval [0,1] and

$$\pi_p = \overline{P}_p^{ENGY} \quad p = 1$$
$$\pi_p = \overline{P}_p^{ENGY} / \overline{P}_{p-1}^{ENGY}, \quad p = 2, ..., P.$$

With this definition of ϕ_p^{ENGY} it is easy to show that $E[\phi_p^{ENGY}] = \overline{P}_p^{ENGY}$, $p = 1, ..., P$. Furthermore, the conditional probability to observe the prices associated with a child node (for the next set of periods) from any given node is $1/N_b$. For $p = 1, ..., \lfloor 3P/4 \rfloor$, the prices deviate from the average through a multiplicative process based on different volatility parameters. Then, for $p = \lfloor 3P/4 \rfloor + 1, ..., P$ the process becomes deterministic and converges toward the average at rate α (the lower is α, the faster is the convergence). Clearly, the correlation between electricity prices in consecutive periods is high, but this is exactly what is observed in real markets. This is the same for the "return toward the average" that is typically observed at the end of the day. For the two reserve types "10 minutes spin" and "10 minutes non spin", deterministic prices are used, that is, $\phi_p^{10S} = \overline{P}_p^{10S}$, $\phi_p^{10N} = \overline{P}_p^{10N}$, $p = 1, ..., P$.

To associate MCPs with the nodes in the scenario tree, the number of generated price vectors N_v was set to $100 \cdot {N_b}^{N_s}$ (see Section 3). This ensures that the MCPs associated with the leaves correspond to averages based on a representative subset of 100 price vectors. For example, the values $N_b = 2$ and $N_s = 2$ correspond to a tree of depth 2, where each internal node has two children. Consequently, from $100 * 2^2 = 400$ price vectors, two subsets of 200 price vectors are associated with the two children nodes of the root and are used to compute an average vector that corresponds to the MCPs at these nodes. Then, four (sub)subsets of 100 price vectors are associated with the four leaves and are used again to compute their MCPs.

For each period p and node $s \in S_p^{int}$, we have $N_{p,s}^{ENGY} = N_b + 1$. Thus, the number of alternative bid choices at a node corresponds to the branching factor plus one. Basically, for each period p and node $s \in S_p^{int}$, the bid prices $BID_{p,s,b}^{ENGY}$, $b = 1, ..., N_{p,s}^{ENGY}$, are selected as follows: the first bid is set at 0, the second bid is the average of the two MCPs associated with the first and second child, the third bid is the average of the two MCPs associated with the second and third child,..., and the last bid is set to a large value, namely, the maximum price observed while constructing the scenario tree. When the bidding price is 0, the bid will automatically fall under the MCP and the associated quantity of electricity will be produced (as in the original stochastic model without bids). Conversely, when the bidding price is very high, the electricity is unlikely to be produced, except in very particular circumstances (see the addition of spikes in Section 4.5).

The three models were implemented in CPLEX within a C++ framework. The maximal size of the CPLEX search tree was limited to 1024 MB. All numerical tests were executed on a machine with a 3 GHz Pentium IV processor and 1 GB of memory.

4.2 Increasing the number of periods

These experiments analyze the impact of the number of planning periods (where each period corresponds to one hour) on the CPU times. In the case of the two stochastic models, the parameters for generating the prices were set to $a_1 = 1.1$, $a_2 = 1.2$, $a_3 = 1.15$, $\alpha = 2$ and $Q = 0.3$. These values were chosen because they best fit the data available to us. The trees of scenarios were constructed with $N_b = 3$ and $N_s = 4$. Note that a CPU time limit of 7200 seconds, or 2 hours, was enforced in all experiments. In Table 1, the results obtained with the deterministic and the stochastic models using an increasing number of periods are shown. The CPU time is indicated as well as the (relative) gap, which is the difference between the upper bound obtained by relaxing the integrality constraints and the best integer solution found, divided by the upper bound. A gap of zero thus indicates an optimal solution.

We note that the stochastic model with bids is the most computationally expensive, due to a larger number of variables. We found optimal solutions for instances with up to 24 periods and suboptimal

	Deterministic		Stochastic without bids		Stochastic with bids	
P	Time (s)	Rel. gap (%)	Time (s)	Rel. gap (%)	Time (s)	Rel. gap (%)
12	0.06	0	0.89	0	25.4	0
24	0.11	0	119.9	0	3652	0
48	0.29	0	4921	0	7200	0.00083
96	3.23	0	5136	0.00221	7200	0.02080
192	12.4	0	7200	0.15709	7200	2.98074
384	32.8	0	7200	0.03997	-	-
768	490	0	-	-	-	-
1536	7200	0.0002	-	-	-	-
3072	7200	0.00003	-	-	-	-

Table 1: Increasing the number of periods P

solutions (i.e., best feasible integer solutions obtained within the alloted time) for instances with up to 192 periods. Over that, the program stopped due to insufficient memory. The stochastic model without bids is a little bit less demanding, while the deterministic model is clearly less computationally expensive.

4.3 Branching factor and number of levels

These experiments study the impact on solution quality of the branching factor and number of levels in the tree of scenarios for the two stochastic models. Solution quality was evaluated through a Monte Carlo approach. First, we generated optimal solutions with CPLEX with the two models for trees of scenarios of different shapes by using different values for parameters N_b and N_s. Then, each solution obtained (i.e., production or bidding decisions) was evaluated on 100,000 new price vectors produced with our stochastic price generation scheme. We recall that a MCP is stored at each node in the tree of scenarios for each period in the corresponding set of periods. This MCP is an average based on a subset of price vectors (see Section 3). Considering only the last period in the period set, which is the most important one with regard to the MCPs that will be observed in the next set of periods, the maximum price value is taken over the subset of price vectors associated with each node. The latter values are then used to follow a path in the tree of scenarios. Let us assume, for example, that we are at a given node a and that nodes b, c and d (from smallest to largest prices) are the three children nodes associated with period set $j \in N_s$. We denote the maximum price value of node $n = a, b, c$ by P_n^{max} and the price of period p_j in some newly generated price vector by $P_{p_j}^{new}$. Then, from node a, we go to:

- node b if $P_{p_j}^{new} \leq P_b^{max}$
- node c if $P_b^{max} < P_{p_j}^{new} \leq P_c^{max}$
- node d otherwise.

Tables 2 and 3 report the results obtained with the two stochastic models with different values of N_b and N_s. In these experiments, an horizon of $P = 24$ periods was used and the parameters for generating the prices were set to $a_1 = 1.1$, $a_2 = 1.2$, $a_3 = 1.15$, $\alpha = 2$ and $Q = 0.3$. The numbers in the tables are averages based on 100,000 different price vectors and correspond to the ratio of the average solution value obtained with a given set of parameters to the best overall average solution value. The latter was obtained with the stochastic model without bids, with $N_b = 5$ and $N_s = 5$. In all cases, the computation was stopped as soon as the relative gap dropped below 0.01%, which was achieved in less than one hour and a half, even on the largest instances. There are two empty entries in Table 3 because the program stopped without returning a solution, due to excessive memory requirements. Table 4 summarizes the results by showing the improvement of the model with bids over the model without bids. This is calculated as follows:

$$1 - \left(\frac{\text{solution value without bids}}{\text{solution value with bids}} \right).$$

When $N_b = 1$ or $N_s = 1$ the results correspond to those that would have been obtained with the deterministic model. The small differences observed between the two stochastic models in these cases can thus be attributed to random noise. Otherwise, for the same values of N_b and N_s (i.e., for the same tree

$N_b \setminus N_s$	1	2	3	4	5
1	.9208	.9207	.9211	.9212	.9208
2	.9210	.9685	.9698	.9806	.9901
3	.9209	.9763	.9806	.9877	.9955
4	.9212	.9770	.9808	.9913	.9988
5	.9206	.9779	.9818	.9935	1

Table 2: Stochastic model without bids: ratio to best solution found

$N_b \setminus N_s$	1	2	3	4	5
1	.9208	.9206	.9210	.9208	.9208
2	.9206	.9730	.9745	.9852	.9900
3	.9210	.9797	.9850	.9913	.9986
4	.9208	.9810	.9894	.9931	
5	.9212	.9817	.9902	.9954	

Table 3: Stochastic model with bids: ratio to best solution found

$N_b \setminus N_s$	1	2	3	4	5
1	0.0000	-0.0001	-0.0002	-0.0005	0.0000
2	-0.0004	0.0046	0.0049	0.0047	-0.0001
3	0.0000	0.0034	0.0045	0.0037	0.0031
4	-0.0005	0.0041	0.0087	0.0019	
5	0.0006	0.0038	0.0084	0.0019	

Table 4: Improvement of model with bids over model without bids

of scenarios) the model with bids is superior, if we except one entry where the two models provide similar results. This improvement reaches a maximum of 0.87%. It is important to compare the two models on the same tree of scenarios, because the latter is an approximation of the underlying real-world process. That is, when N_b and N_s increase, the tree gets larger and a better approximation is obtained.

Given that the stochastic model without bids is less computationally demanding, it was possible to solve it using a tree of scenarios with $N_b = 5$ and $N_s = 5$ (as shown in the next subsection, we were lucky to get a solution, given that the model cannot be solved with most parameter values). This model produces an average solution value that improves by 0.14% the best average value achieved with the model with bids. In the latter case, this best result was obtained with $N_b = 3$ and $N_s = 5$, because it was not possible to solve larger models. In the next subsection, we will see that the best solution values that can be obtained by integrating bids (even if these solutions are produced with less detailed models) are superior to those that can be obtained without bids, when price volatility is increased and when spikes are introduced.

4.4 Increased volatility

By increasing price volatility (variance) or by adding spikes, the environment becomes less predictable and a good bidding strategy is likely to provide substantial benefits. The following experiments compare the two stochastic models with regard to price volatility by gradually increasing the values of parameters a_1, a_2 and a_3. The other parameters for generating the stochastic prices were set at their usual values $\alpha = 2$ and $Q = 0.3$. In all cases, the computation was stopped as soon as the relative gap dropped below 0.01%, which was achieved in less than one hour and a half, even on the largest instances. Table 5 shows the results with a tree of scenarios generated with $N_b = 3$ and $N_s = 4$ over an horizon of $P = 24$ periods (i.e., a model of moderate size). Each entry in the table is the ratio of the average solution value obtained with a given set of volatility parameters to the best average value (obtained with the model with bids and parameter values $a_1 = a_2 = a_3 = 1.5$). The improvement of the model with bids over the model without bids is also shown in the table. Table 6 shows the same results, but with the largest models that could be solved. As we can see, for many values of a_1, a_2 and a_3, the model without bids with $N_b = 5$ and $N_s = 5$ does not provide a solution. We thus show the results with $N_b = 4$ and $N_s = 5$ in a separate column. For

the model with bids, a solution is always obtained with $N_b = 3$ and $N_s = 5$. These results clearly show that the improvement obtained by integrating a bidding strategy increases with price volatility.

$a_1 = a_2 = a_3$	Without Bids	With Bids	Improvement
1.00	.692	.692	.000
1.05	.701	.701	.000
1.10	.722	.722	.001
1.15	.747	.748	.001
1.20	.776	.782	.008
1.25	.807	.815	.010
1.30	.840	.853	.015
1.35	.874	.893	.021
1.40	.903	.929	.028
1.45	.934	.967	.034
1.50	.965	1	.035

Table 5: Increased volatility: Improvement of model with bids over model without bids with $N_b = 3$ and $N_s = 5$.

$a_1 = a_2 = a_3$	Without Bids $N_b = 5$	Without Bids $N_b = 4$	With Bids	Improvement
1.00	-	.680	.680	.000
1.05	-	.690	.690	.000
1.10	-	.712	.712	.000
1.15	-	.739	.740	.001
1.20	-	.769	.772	.003
1.25	-	.801	.808	.008
1.30	-	.835	.845	.011
1.35	.872	.870	.883	.012
1.40	.907	.905	.921	.016
1.45	.942	.940	.961	.020
1.50	.977	.975	1	.023

Table 6: Increased volatility: Improvement of model with bids with $N_b = 3$ and $N_s = 5$ over model without bids with $N_b = 4, 5$ and $N_s = 5$.

4.5 Spikes

Spikes are observed in deregulated markets when some unexpected events happen, like a power plant break down or a sudden demand outburst. To add spikes, we first generate prices according to the stochastic process introduced in Section 4.1. Then, a spike is added with probability P_{spike}. A single period is chosen for this purpose, and is selected according to a random uniform distribution. The height of the spike is H_{spike} times the maximum price observed over all periods. Clearly, when $H_{spike} > 1$, we obtain a price spike that is higher than all other observed prices.

The complete procedure is thus the following. First, we create a tree of scenarios based on prices that are generated with the usual stochastic process of Section 4.1. An optimal solution is then obtained with CPLEX. Finally, this solution is evaluated with newly generated prices using the modified procedure with the (possible) addition of a spike.

In the experiments reported here, the parameters for the stochastic process were set to $a_1 = 1.1$, $a_2 = 1.2$, $a_3 = 1.15$, $\alpha = 2$ and $Q = 0.3$, while $P = 24$ periods were considered. As usual, the computation was stopped as soon as the relative gap dropped below 0.01%.

Table 7 shows the improvement obtained with the stochastic model with bids over the one without bids, based on a tree of scenarios with $N_b = 3$ and $N_s = 4$ (i.e., a model of moderate size). Table 8 shows the same improvement, but with the largest models that could be solved (i.e., with $N_b = 3$, $N_s = 5$ for the model with bids and $N_b = 5$, $N_s = 5$ for the model without bids).

In Table 7, we see that the improvement obtained with the model with bids increase with the probability of a spike. The same behavior is also observed when the height of the spike increases. Thus, for a

$P_{spike} \setminus H_{spike}$	1.5	2	3	4
0.1	0.0041	0.0042	0.0046	0.0050
0.2	0.0044	0.0048	0.0056	0.0063
0.3	0.0048	0.0053	0.0065	0.0076
0.4	0.0051	0.0059	0.0074	0.0089
0.5	0.0055	0.0064	0.0082	0.0101
0.75	0.0064	0.0077	0.0105	0.0132
1	0.0073	0.0090	0.0126	0.0160

Table 7: Addition of spikes: Improvement of model with bids over model without bids with $N_b = 3$ and $N_s = 4$.

$P_{spike} \setminus H_{spike}$	1.5	2	3	4
0.1	-0.0003	-0.0001	0.0001	0.0004
0.2	0.0001	0.0004	0.0010	0.0016
0.3	0.0004	0.0007	0.0015	0.0023
0.4	0.0007	0.0012	0.0022	0.0033
0.5	0.0010	0.0017	0.0030	0.0044
0.75	0.0016	0.0026	0.0044	0.0063
1	0.0024	0.0037	0.0063	0.0089

Table 8: Addition of spikes: Improvement of model with bids with $N_b = 3$ and $N_s = 5$ over model without bids with $N_b = 5$ and $N_s = 5$.

given tree of scenarios, the model with bids is clearly superior. The same is observed in Table 8, even if the model with bids is now based on a less detailed tree of scenarios. The only exception is when P_{spike} and H_{spike} are at their lowest values in the table. In the next subsection, we will now see what happens when spikes are added to slightly increased volatile prices.

4.6 Increased volatility and spikes

This section presents results obtained with a combination of spikes and increased volatility. Basically, the value of each volatility parameter was increased by 0.05, that is, from $a_1 = 1.1$, $a_2 = 1.2$, $a_3 = 1.15$ to $a_1 = 1.15$, $a_2 = 1.25$, $a_3 = 1.2$. The other parameter values were kept the same.

Table 9 shows the improvement obtained with the stochastic model with bids over the one without bids, based on a tree of scenarios with $N_b = 3$ and $N_s = 4$. Table 10 shows the same improvement, but with the largest models that could be solved (i.e., using $N_b = 3$, $N_s = 5$ for the model with bids and $N_b = 5$, $N_s = 5$ for the model without bids).

$P_{spike} \setminus H_{spike}$	1.5	2	3	4
0.1	0.0098	0.0102	0.0109	0.0116
0.2	0.0105	0.0112	0.0126	0.0140
0.3	0.0112	0.0123	0.0143	0.0164
0.4	0.0119	0.0133	0.0159	0.0186
0.5	0.0126	0.0142	0.0175	0.0208
0.75	0.0143	0.0168	0.0216	0.0263
1	0.0160	0.0192	0.0254	0.0314

Table 9: Increased volatility and addition of spikes: Improvement of model with bids over model without bids with $N_b = 3$ and $N_s = 4$

We can see that by combining increased volatility with spikes, the trends observed in Section 4.5 are more obvious. In particular, the model with bids is now better over all values of P_{spike} and H_{spike}. Also, the largest improvements observed now reach 3.14% and 1.66% in Tables 9 and 10, respectively (as compared with 1.6% and .89% in Tables 7 and 8).

$P_{spike} \setminus H_{spike}$	1.5	2	3	4
0.1	0.0023	0.0025	0.0029	0.0033
0.2	0.0029	0.0034	0.0043	0.0052
0.3	0.0034	0.0040	0.0052	0.0063
0.4	0.0039	0.0047	0.0063	0.0078
0.5	0.0044	0.0055	0.0076	0.0096
0.75	0.0056	0.0071	0.0099	0.0126
1	0.0070	0.0090	0.0129	0.0166

Table 10: Increased volatility and addition of spikes: Improvement of model with bids with $N_b = 3$ and $N_a = 5$ over model without bids with $N_b = 5$ and $N_a = 5$.

5 Conclusion

This paper has introduced a stochastic programming model for hydroelectricity production that integrates strategic bidding in a deregulated market. It was developed to help price-taker hydroelectricity producers in their bidding tasks. The model is realistic, as it takes into account important physical, operational and financial constraints and characteristics of the application under study. Numerical results show that this new model is superior to a previous model that did not integrate bids. This superiority is enhanced when price volatility is increased or when spikes are added. In the latter case, the best solutions achieved with the model with bids outperformed the best solutions achieved with the model without bids, even if these solutions were obtained with less detailed trees of scenarios (due to computational limitations).

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